Nondestructive Measurement of Complex Permittivity for Dielectric Slabs

MARC C. DECRÉTON, MEMBER, IEEE, AND MUNIKOTI S. RAMACHANDRAIAH, MEMBER, IEEE

***Abstract*—A method has been developed for the precise nondestructive measurement of the dielectric constant and losses of slab-like samples such as microstrip substrates, for instance. Basically, the test setup consists of an open-ended rectangular waveguide, the flange of which is placed in contact with one side of the dielectric material, the other one being backed by a metal plate. The waveguide can be either simply cut at its end, or terminated by an inductive or capacitive iris.**

The reflection characteristics or the resonance parameters are related to the real and imaginary parts of the permittivity by means of computer-generated charts or an optimization program.

I. INTRODUCTION

The increasing use of microwaves for industrial testing procedures has recently generated a strong need for accurate nondestructive techniques for the measurement of permittivity at microwave frequencies. Properties such as moisture content of materials or dielectric constant, and local inhomogeneities in microstrip substrates are often important factors which can be evaluated and controlled efficiently with microwaves. Most classical methods to measure material dielectric properties require cutting and machining of samples to close tolerances [1], [2]. Besides the cost and delays necessary for such operations, these destructive techniques are quite unsuited for on-line control of industrial processes.

A method of measurement has been presented recently for thick or highly lossy materials in which the reflections from the back end can be neglected [3]. This approach is extended here to materials in slab form. It is particularly useful for the determination of the dielectric properties of microstrip substrates but also finds applications in a wide range of industrial testing processes (the moisture content of sheet materials for instance).

The basic test setup consists of an open-ended waveguide sensor which is terminated by a large metallic flange (Fig. 1). This flange is placed in firm contact with the slab of material to be tested, which in turn is backed by a flat conducting plate. The aperture may be either the full waveguide cross section or a smaller rectangular opening such as an inductive iris, e.g., symmetrically located with respect to the waveguide axis. This waveguide discontinuity can be represented by an equivalent admittance Y which is a function of the complex permittivity of the sample $\epsilon_r = \epsilon_r' - j\epsilon_r''$, its thickness t and the aperture area S

$$Y = Y(\epsilon_r', \epsilon_r'', t, S). \tag{1}$$

This admittance Y can be measured in two different ways. The first way is to determine the amplitude and phase of the reflected wave with respect to the one produced by a short circuit placed on the flange. The second method is to consider a resonant waveguide cavity terminated at one side by the flanged open-ended waveguide (with a small iris, to increase the Q factor) and a coupling hole at the other side. The resonant frequency and the Q factor of the cavity are then measured.

In both cases, the measured values can be related to the equivalent admittance Y and hence to the permittivity of the slab by computer-generated charts or with an optimization program.

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The authors are with the Chair of Electromagnetism and Microwaves, Ecole Polytechnique Fédérale de Lausanne, Lausanne CH-1007, Switzerland.

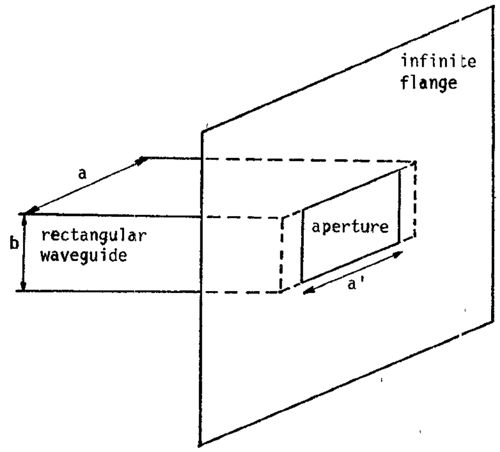

Fig. 1. Open-ended rectangular waveguide.

II. THEORETICAL DETERMINATION OF THE INPUT ADMITTANCE

Much work has been done on aperture antennas and flanged open structures. Lewin [4] and Compton [5] developed a variational principle for the admittance of these structures and some applications have been published in recent years [6]–[8]. In [9], the study was extended to the inverse problem where the material properties are computed from the admittance measurements.

In most of these reports, one uses a Fourier transform analysis in order to solve the integral equation. This introduces infinite integrals in the complex plane and a relatively long and intricate numerical computer program. For simple shapes, such as the rectangular aperture considered here, it is more efficient to solve the problem directly, making use of the variational property of the admittance expressions [6].

This admittance Y for the flanged open-ended rectangular waveguide can be written as [3], [13]

$$Y = \frac{abY_0}{2\Gamma_0} \frac{1}{D^2} \int_S \int_S E_y(x,y) E_y(\xi,\eta) K(x,y;\xi,\eta) \, dS \, dS \tag{2}$$

with

$$K = \left(\frac{\partial^2}{\partial x^2} + k^2\right) G + \sum_{i=1}^{\infty} \frac{4\epsilon_i}{ab\Gamma_i} T_i \left\{\left(\frac{m_i\pi}{a}\right)^2 - k^2\right\}$$

$$T_i = \sin\frac{m_i\pi x}{a} \sin\frac{m_i\pi\xi}{a} \cos\frac{n_i\pi y}{b} \cos\frac{n_i\pi\eta}{b}$$

$$D = \int_S E_y(\xi,\eta) \sin\frac{\pi\xi}{a} \, d\xi \, d\eta$$

$$\Gamma_i^2 = \left(\frac{m_i\pi}{a}\right)^2 + \left(\frac{n_i\pi}{b}\right)^2 - k^2$$

where

$$\epsilon_i = 1 \quad \text{if} \quad n_i \neq 0$$

and

$$\epsilon_i = 1/2 \quad \text{if} \quad n_i = 0$$

where

Y_0 dominant TE_{10}-mode admittance;

a,b	dimensions of the rectangular waveguide;
E_y	the transverse electric field in the aperture plane;
S	aperture area;
k	free-space wavenumber;
(m_i,n_i)	indices for the modes excited in the waveguide.

The factor G appearing in (2) is the Green's function valid in the dielectric medium outside the waveguide (Fig. 2). This Green's function relates the electric field to the magnetic field in the aperture plane through the following equation:

$$H_x(x,y) = \int_S E_y(\xi,\eta)\left(\frac{\partial^2}{\partial x^2} + k^2\right) G(x,y;\xi,\eta)\, d\xi\, d\eta. \tag{3}$$

In the case of a dielectric sample of infinite thickness, the free-space Green's function can be used [3]

$$G = \frac{\exp(-jk_r r)}{r} \tag{4}$$

where

$$k_r = \omega(\epsilon_0\epsilon_r\mu_0)^{1/2}$$

and

$$r = ((x-\xi)^2 + (y-\eta)^2)^{1/2}.$$

The metal back of the dielectric slab introduces a series of successive reflections. The flange is actually considered as a metal plate with the aperture replaced by a source distribution [6]. The composite Green's function is constructed by means of the successive images of the source along the z direction [10], [13]

$$G = \sum_{n=-\infty}^{+\infty} \frac{1}{r_n} \exp(-jk_r r_n) \tag{5}$$

with r_n being the distance from the nth image to the observation point (x,y)

$$r_n = ((x-\xi)^2 + (y-\eta)^2 + (2tn)^2)^{1/2}.$$

For the particular case of $t = \infty$, the expression of Y in (2) becomes the one obtained in [3]. It is also very similar to the variational formulas given by Collin [11] for the cases of obstacles and discontinuities in rectangular waveguides.

The variational properties of (2) suggest a solution by a Rayleigh–Ritz approximation, the unknown function $E_y(x,y)$ being expanded over the complete set of the orthonormal waveguide modes. For the case of a waveguide simply cut at its end without an iris we have

$$E_y = \sum_{i=0}^{N} A_i \sin\frac{m_i\pi x}{a} \cos\frac{n_i\pi y}{b} \tag{6}$$

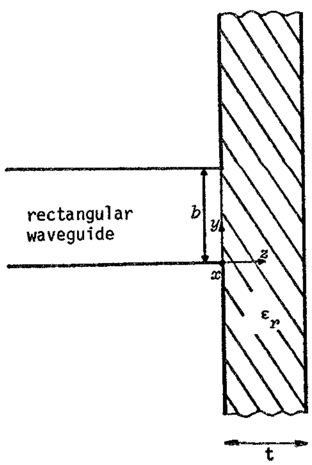

Fig. 2. Experimental setup and coordinate system.

with N the number of waveguide modes to be considered. This approach allows one to refine the approximation to the desired accuracy by taking more test functions, thus extending the two-term series used by Galejs [6]. For the case of an iris-terminated waveguide, the efficiency of the method is greatly improved by choosing a set of functions more appropriate to the particular geometry of the aperture. For instance, for an inductive iris of aperture (a',b) where $a' < a$, one simply replaced a by a' in (6) thus considering the modes of a rectangular waveguide having the aperture as cross section.

The usual Rayleigh–Ritz technique yields a set of linear simultaneous equations. Solving it for the unknowns A_i and replacing (6) into (2) gives the value of the input admittance Y.

III. REFLECTION COEFFICIENT AND RESONANCE PARAMETERS

Knowing the equivalent admittance Y, determination of the reflection coefficient at the discontinuity is immediate. For the first measurement approach, the measured quantities are the VSWR and the phase shift ϕ with respect to a short circuit placed directly on the flange. For the second approach, we consider a cavity of length l formed by a section of rectangular waveguide terminated by the admittance Y at one end and a coupling iris at the other end (Fig. 3). The coupling iris has an equivalent admittance Y_{iris} which can be calculated by the formulas given by Marcuvitz [12] for the iris with a symmetric circular aperture. This allows one to determine the input admittance of the cavity, the resonance being obtained when Im $(Y_{in}) = 0$. The loaded Q factor is computed from the two half-power frequencies f_1 and f_2

$$Q_L = \frac{f_r}{f_1 - f_2}. \tag{7}$$

IV. EXPERIMENTAL VERIFICATION

As a checkup for the computer program, some experiments were performed. An example is shown in Fig. 4 where the resonance characteristics are given for an iris-terminated waveguide structure loaded with Perspex ($\epsilon_r = 2.56$, $\tan\delta = 0.0008$) slabs of different thicknesses. The computed Q takes into account the coupling iris and the wall losses. Both the experimental and theoretical results are presented and the correspondence between theory and measurements is satisfactory.

The number of successive reflections considered in the computer program (5) is ten in the average, but more terms are needed for good convergence when the thickness is very small.

V. NUMERICAL TREATMENT AND DETERMINATION OF THE PERMITTIVITY

Equation (2) is not analytically reversible and there is no simple formula relating ϵ_r to the measured quantities as is the case, for instance, with the "infinite sample method" [2]. An indirect way must be used to determine the permittivity. The calculation proce-

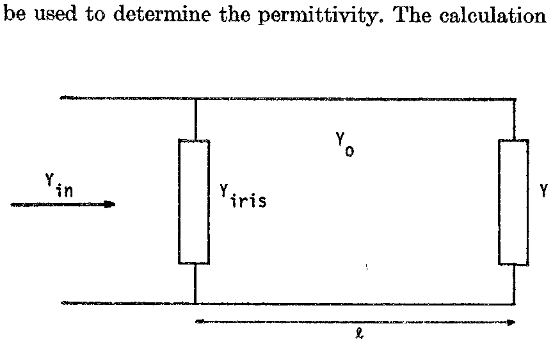

Fig. 3. Equivalent circuit of the waveguide cavity.

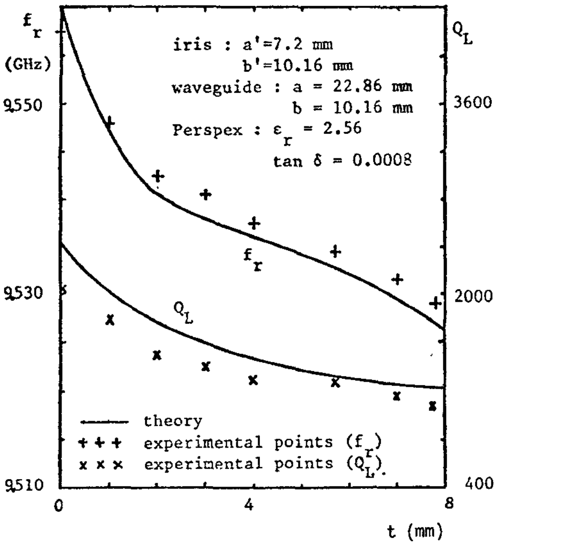

Fig. 4. Resonant parameters of the cavity loaded with Perspex sheets (ϵ_r = 2.56, tan δ = 0.0008, WR-90 waveguide, a' = 1.79 cm, TE_{107} mode).

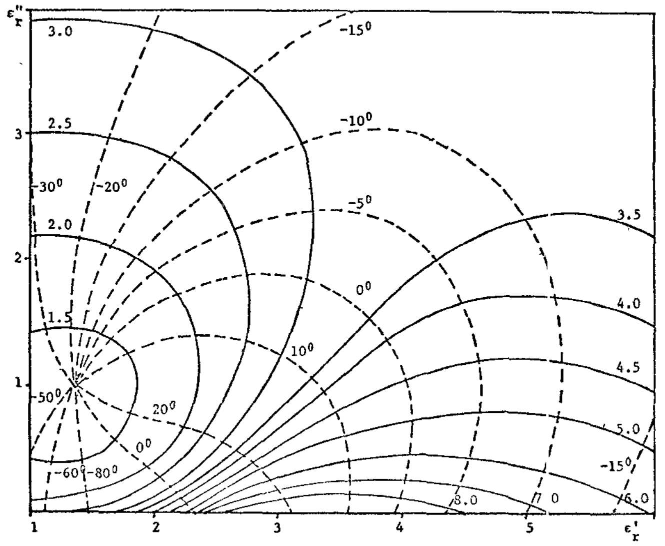

Fig. 5. Chart relating the VSWR (solid lines) with the permittivity (f = 10 GHz, WR-90 waveguide).

dure can be successively applied to different permittivities of the dielectric slab for some given waveguide size and operating frequency. Computer-generated charts can thus be elaborated where the measured parameters (amplitude and phase, or resonant frequency and Q factor) are drawn as a function of ϵ_r' and ϵ_r''. For the amplitude and phase method, the particular case of the WR-90 waveguide (X band) operated at 10 GHz yields the graph of Fig. 5. Knowing the VSWR and the phase angle ϕ of the reflected wave (as compared to the reflection due to a short circuit), the real and imaginary parts of the permittivity can be obtained directly from this chart. Similar charts can be drawn for f_r and Q_L.

This procedure may be used satisfactorily when measurements are consistently carried out at the same frequency f_r in the same waveguide or with the same cavity length and coupling iris. However, it is often required in practice to carry out measurements with different parameters (waveguide size, frequency, etc.), and a great number of charts would become necessary, the method then becoming impractical. For this reason, a technique similar to the one described in [3] is used here. The computer program is completed by a search loop in which the measured VSWR and ϕ are introduced. The computer then finds, by means of an optimization process similar to the "razor search" [14], the corresponding value of the complex permittivity ϵ_r.

This method has been used to determine the permittivity of samples of specific materials. The results are shown on Table I. They are compared with a classical technique: the short-circuited sample method described in [1, pp. 503–508]. Method I in Table I refers to this classical technique and Method II is the open-ended waveguide technique. The sample thickness is t in both cases and Δ refers to the difference between Method I and Method II. The optimization program is used with an average of 40 evaluations of the objective function. One observes a good correspondence especially for the real part of the permittivity. On the other hand, the losses are given with a 10-percent error. This is partly due to differences between the theoretical model and the experimental reality. The samples and the flange are not infinite in the x and y directions as it is assumed in the theoretical analysis. Parasitic reflections at the edges of the sample reduce the accuracy of the measurement, the effect becoming significant if small samples are used. From field calculations in the dielectric slab and experimental tests, the flange and sample dimensions were chosen as 10 × 10 cm with an X-band waveguide. This represents a satisfactory compromise between the simulation of ideal conditions for precision and the availability of the samples, their preparation, and the compatibility with the measurement setup dimensions.

TABLE I
Experimental Determination of the Permittivity of Different Materials

Name of material	Method I (short-circuited sample)			Method II (open-ended waveguide)				
Emerson & Cuming	t cm	ϵ_r	$\tan\delta$	t cm	ϵ_r	$\tan\delta$	$\Delta\epsilon_r$ %	$\Delta\tan\delta$ %
Stycast K4	0.650	4.10	<0.005	0.650	4.17	0.0025	1.7	-
Stycast K9	0.270	9.08	<0.005	0.656	9.02	0.0031	0.7	-
Stycast K20	0.183	18.96	<0.005	0.659	18.78	0.0040	1.0	-
HF2050	0.262	10.52	0.108	0.670	10.19	0.119	3.1	9.2
HF1000	0.211	15.21	0.234	0.673	15.42	0.210	1.4	10.2

A minimum thickness of the order of 0.5 mm is necessary for measurement of lossless dielectrics at X-band frequencies, as the number of reflections which must be considered in the computer program is inversely proportional to thickness. For very thin samples, computer time increases quite substantially and cost may become prohibitive.

Another source of error lies in the inaccuracy in the measurement of the phase shift ϕ. With usual measurement techniques, it is typically of the order of a few degrees, and a careful measurement with a slotted line equipped with a precision dial gauge still leaves an error of the order of 1°. This inaccuracy can be reduced by using the cavity approach where the measured quantities are the resonance frequency and the Q factor, both of which can be determined experimentally with a comparatively greater accuracy. On the other hand, the latter technique does not allow a free selection of frequency: the measurement can only be performed at the resonant frequencies of the loaded waveguide section. This, however, is a drawback only for highly frequency-sensitive materials. Further details of the measurements by the resonant technique are available in another publication [15].

VI. CONCLUSION

The equivalent terminal admittance of an open-ended rectangular waveguide loaded with a metal-backed dielectric slab has been theoretically calculated. The complex reflection coefficient of the waveguide and the characteristics of a resonant waveguide section terminated by this admittance have been evaluated. Their correspondence with experimental data is found to be quite satisfactory. This allows one to set up a measurement technique which is nondestructive and needs only two measurements (VSWR and phase shift, or resonance frequency and Q factor). The relative permittivity can be calculated by means of computer-aided data processing and time-saving charts can be drawn for particular cases. The reflection coefficient approach allows a measurement of the material properties at all frequencies, limited only by the bandwidth of the waveguide. A good precision is obtained for the dielectric constant but the losses are determined only with an average 10-percent error due to the difficulty in measuring the phase shift. The cavity approach, on the other hand, overcomes this kind of inaccuracy but restricts the measurements to certain frequencies determined by the cavity geometry and the unknown material to be tested.

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Integrated Diode Phase-Shifter Elements for an X-Band Phased-Array Antenna

MARK E. DAVIS, MEMBER, IEEE

Abstract—**The design and production of 502 X-band P-I-N diode phase-shifter elements for a transmissive phased-array radar are presented. These elements consist of three phase-shifter states and two integrated dipole radiators formed using microwave integrated circuit techniques. The detailed design of loaded-line phase shifters and effects of circuit variations during production are examined in terms of measured performance. Finally, the performance of the phase shifters in the phased array is demonstrated through computed and measured antenna patterns giving quantitative results.**

INTRODUCTION

In a phased-array system where high-speed scanning, transmission reciprocity, and array weight are prime engineering concerns, P-I-N diode phase shifters are usually chosen. Considerable effort has been expended in the past ten years to arrive at the optimum design and lowest cost of P-I-N diode phase shifters for phased arrays. Several competitive factors dictate a design in each system that best meets the overall requirements of the phased-array radar. The factors that most contribute to the design of the phase shifter are the available space, coupling to the array radiator, location and complexity of the driver units, and finally, loss. Minimizing loss, while simultaneously optimizing the overall system design for both performance and cost effectiveness, usually becomes the most challenging engineering consideration.

The radiating element in a phased array has historically paced the phased-array performance. Because of the need to optimize the antenna directivity over a large scan volume, minimize mutual coupling effects on pattern gain, and maximize match with the transmitter/receiver circuitry, the radiator and the phase shifter have, in the past, been designed separately. This dictates a transition from the radiating element to the phase shifter and then to the feed distribution system. Due to this necessity of transitioning from one propagation medium to another, the overall design has increased loss due to mismatch reflections in the system. Typically, these transitions can add from 0.5 to 1.0 dB of loss to the element design at X band, depending on the frequency and transmission phase of the element. An even more important consideration than loss, however, is the increase in cost and reduction in reliability presented by having multiple connectors and transitions in the element design. An increased emphasis is placed on the "connectorless" element in phased-array design.

With these considerations, a novel approach to diode phase-shifter-element design was chosen in fabricating a large X-band transmissive phased-array antenna. The basic phase-shifter element is shown in Fig. 1. This design uses a loaded-line phase-shifter approach for the 45° and 90° sections. Loaded-line designs were chosen because of their compatibility with printed-circuit microstrip design and minimizing the number of diodes, and hence diode loss, required for the small phase-shift sizes. Two 45° sections are used in parallel to obtain the 90° desired phase shift. This was chosen in a trade-off of minimum diode loss, reflection loss, and fabrication ease over a hybrid-coupled or switched-line phase-shifter design [1]. For the 180° bit, a new design was chosen that gives an exact 180° of phase shift through coupling the microwave energy into the unbalanced propagation of a slot-guide transmission line etched in the microstrip ground plane [2]. This design will be presented in more detail in the next section of the paper.

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The author is with the Aircraft Equipment Division, General Electric Company, Utica, N. Y. 13503.